Laplace Transform

- what it does?

laplace transform converts the time domain signal to frequency domain signal.

Time domain signal $\longrightarrow$ frequency domain signal.

$f(t) \longrightarrow f(s).$

$$\mathcal{L}[f(t)] \longrightarrow F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt.$$

* 0^- is mandatory because for delta signal or impulse signal 0^- required.

Here $s = \sigma + j\omega$.

Table of Laplace Transform pairs.

	f(t)	f(s)
1.	$u(t)$	$1/s$.
2	t	$1/s^2$
3	e^{at}	$\frac{1}{s-a}$
4	e^{-at}	$\frac{1}{s+a}$
5	$\delta(t)$	1
6	K	K/s
7	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
8	$\cos\omega t$	$s/s^2+\omega^2$

$$\mathcal{L}[u(t)] = \int_{0^-}^{\infty} u(t)\, e^{-st}\, dt \qquad \begin{aligned} u(t) &= 1 \quad t > 0 \\ &= 0 \quad t < 0 \end{aligned}$$

$$= \int_{0^-}^{\infty} e^{-st}\, dt.$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= -\frac{1}{s}\left[0 - 1\right] = \frac{1}{s}.$$

Important properties of Laplace Transform.

1) Linearity $\mathcal{L}[a f(t) + b f_2(t)]$

$$= a f_1(s) + b f_2(s)$$

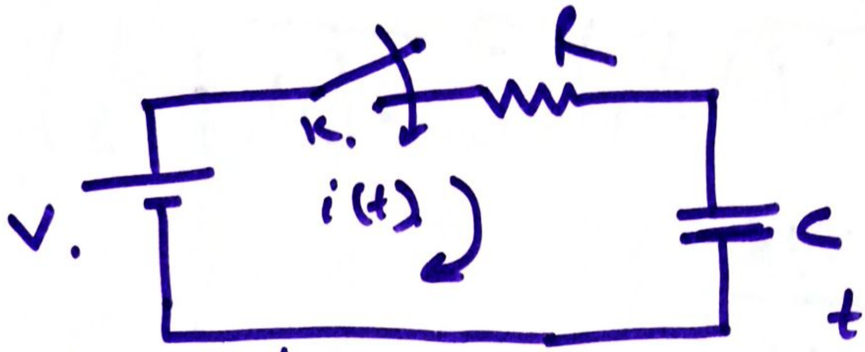
2) Time Diffrentiation

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = s f(s) - f(0^-)$$

3) Time Integration

$$\mathcal{L}\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} f(s)$$

Analysis of R-C. Series circuit using Laplace Transform



when switch is closed at $t=0$.

By K.V.L

$$V u(t) = iR + \frac{1}{C}\int_{-\infty}^{t} i\,dt.$$

By L.T — By Applying property. T.I.

$$\frac{V}{s} = I(s)R + \frac{1}{C}\left[\boxed{\frac{I(s)}{s}} + \frac{q(0^-)}{s}\right]$$

$\frac{q(0^-)}{s}$ = If There was initial voltage.

As $\frac{q}{c}$ = voltage

But There is no initial voltage

so,

$$q(0^-) = 0.$$

$$\cancel{\frac{V}{s} = I(s)R + \frac{1}{Cs}}$$

$$\frac{V}{s} = R\,I(s) + \frac{1}{Cs} I(s)$$

$$= I(s)\left[R + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V}{s\left(R + \frac{1}{Cs}\right)} = \frac{V/R}{s\left[1 + \frac{1}{RCs}\right]}$$

$$I(s) = \frac{V/R}{s + \frac{1}{RC}}$$

By Applying Inverse L.T.

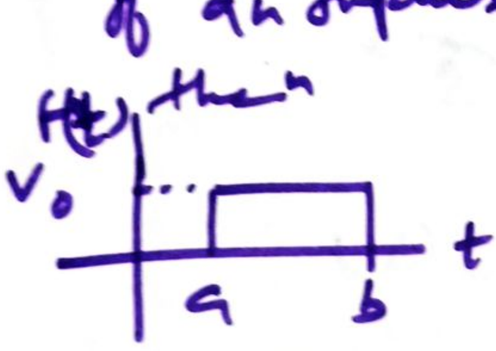
$$i(t) = \frac{V}{R}\left(\frac{1}{s + 1/RC}\right)$$

$\because \frac{1}{s+\frac{1}{RC}}$ is the form of $\frac{1}{s+a}$

$$\boxed{i(t) = \frac{V}{R} e^{-\frac{1}{RC}t}}$$

~~Analysis of R-L circuit~~

If an impulse voltage is Applied $f(t)$ then



$$ut = V_0\left[u_a(t) - u_b(t)\right]$$

then

$$I(s) = \frac{V}{R}\left[u_a(t)e^{-\frac{1}{RC}(t-a)} - u_b(t)e^{-\frac{1}{RC}(t-b)}\right]$$